Accelerating Mathematical Knot Simulations with R on the Web

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December 12, 2016

Content

I Introduction

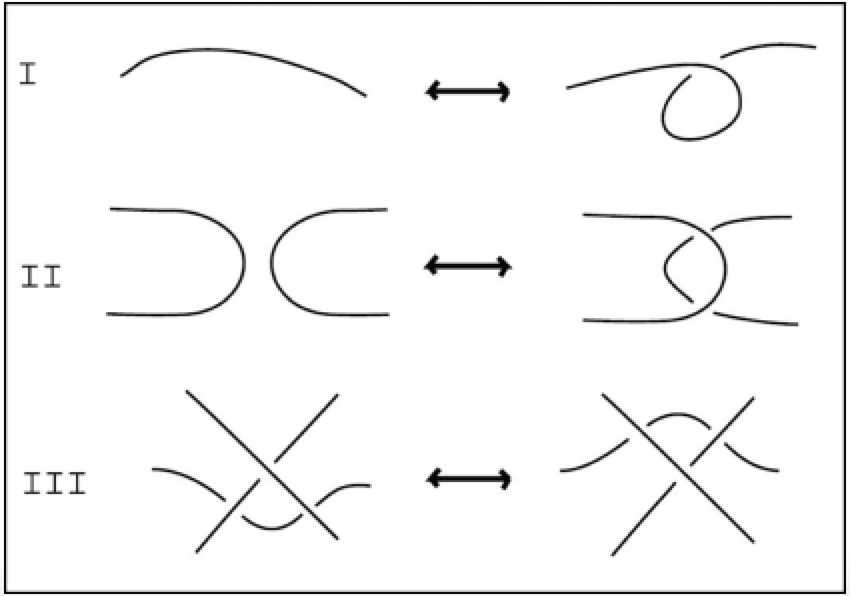
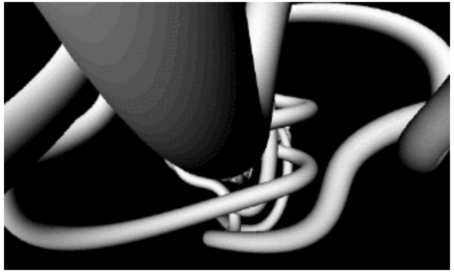


Figure 1. The three Reidemeister moves.



(I) Twist or untwist in either direction.
(II) Move one loop completely over another.
(III) Move a string completely over or under a crossing.

I Introduction

✓ Self-deformable model

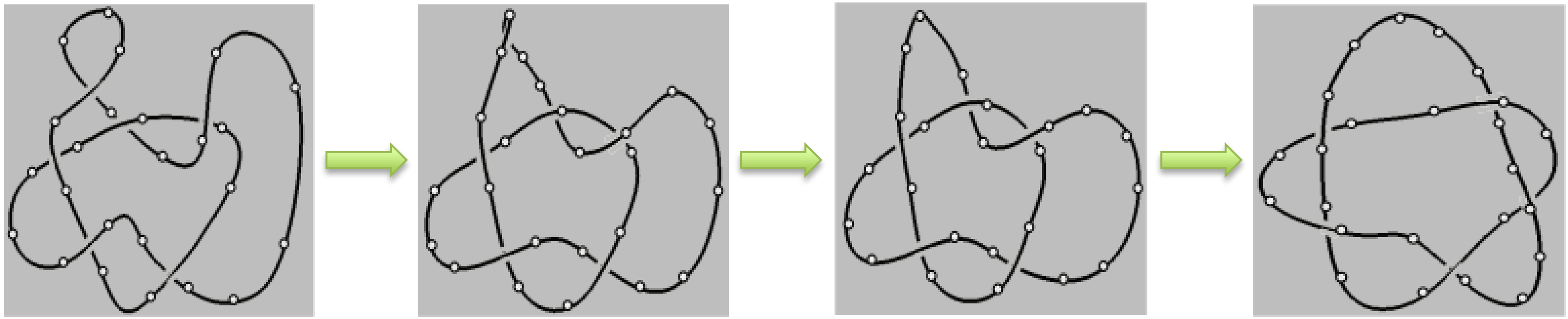


Figure 2. Typical screen images of the self-deformation.

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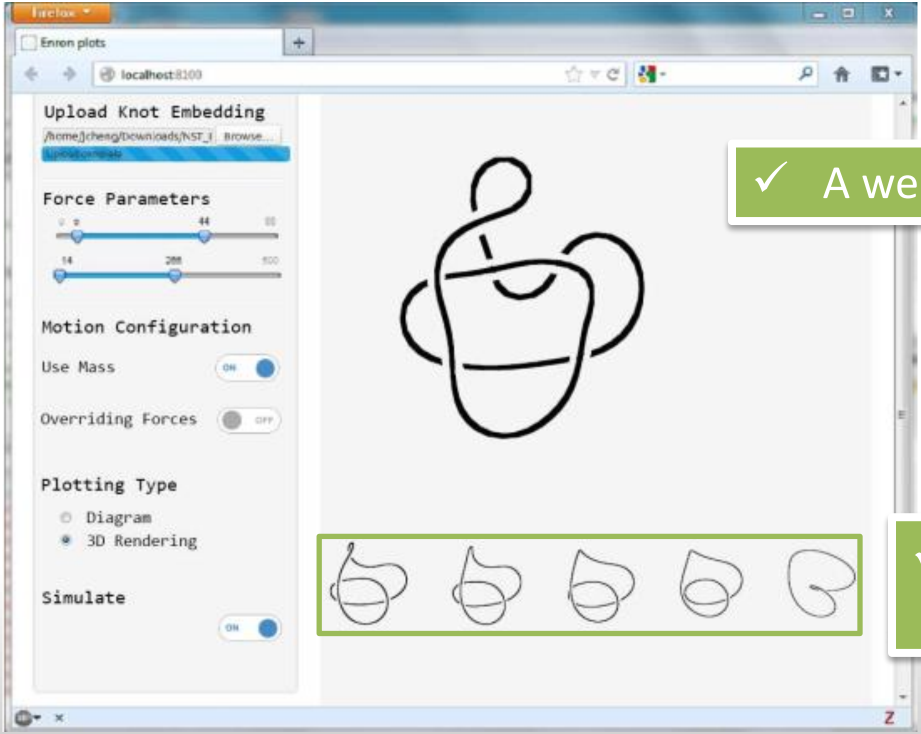
I Introduction

- ✓ Parallelization



- ✓ A web-based interface
- ✓ Self-deformable model

Figure 3. Prototype of MathSimWeb.

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II A New Visualization Paradigm for Exploring Mathematical Knots

✓ Force-directed Algorithm

→ Force Laws for Automatic Topological Refinement

- Attractive Mechanical Force

  $F_m = Hr^{1+\beta}$

  r is the distance between masses

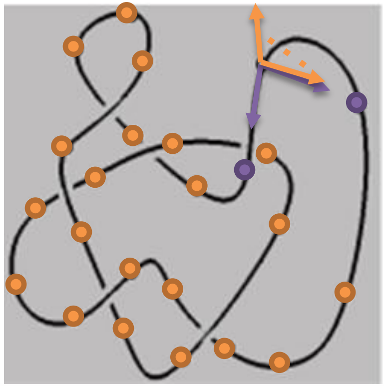
  H is a constant

- Repulsive electrical force:

  $F_e = Kr^{-(2+\alpha)}$

  r is the distance between masses

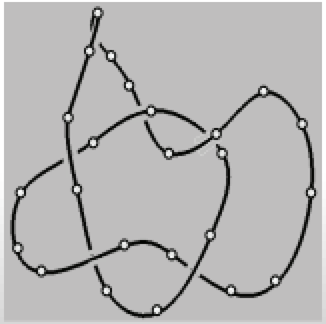
  K is a constant

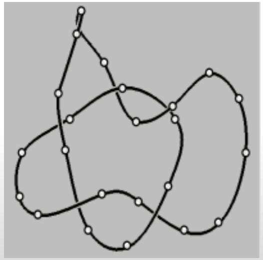


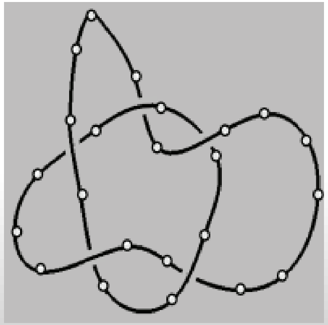
II A New Visualization Paradigm for Exploring Mathematical Knots

✓ Collision Avoidance for Topology Preservation

- Point-segment collision
- Segment-segment collision







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II A New Visualization Paradigm for Exploring Mathematical Knots

✓ Adding Masses to Constrain Deformation in Configuration

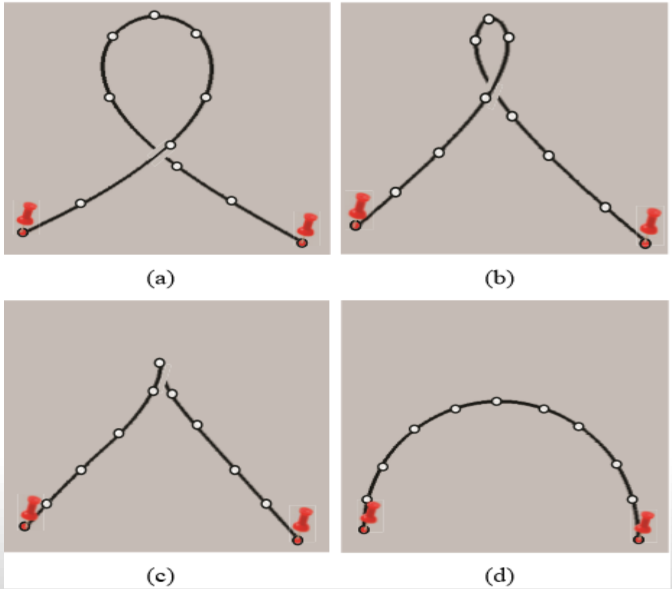

Figure 4. The topological relaxation of a curve.

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II A New Visualization Paradigm for Exploring Mathematical Knots

✓ Adding Overriding Forces to Supplement Unguided Refinement

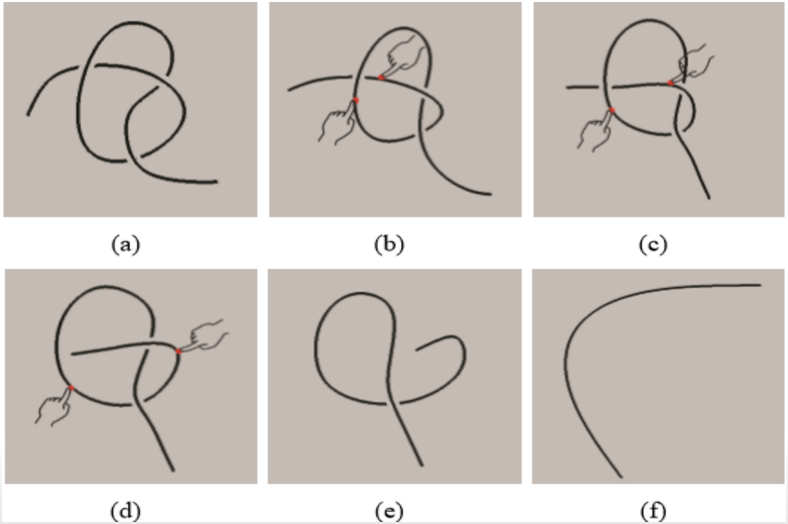

Figure 5. Untying an overhand knot with user-defined overriding forces in conjunction with the self-deformation.

III Accelerating Geometric Deformations Of Complexity

√ Extracting Key Moments for Mathematical Movies

→Through Identifying the minima number of crossing points among all possible 2D projections

IV MathSimWeb — Putting Computation and Visualization Together

```
library(Matrix);
n <- 8192;
X <- Hilbert(n);
A <- nearPD(X);
system.time(B <- chol(A$mat));
# user   system elapsed
# 97.990   0.356   98.591
library(HiPLARM)
system.time(B <- chol(A$mat));
# user   system elapsed
# 1.012   0.316   1.337
```

Figure 3: A brief example to highlight the benefits of using HiPLARM.

IV MathSimWeb — Putting Computation and Visualization Together

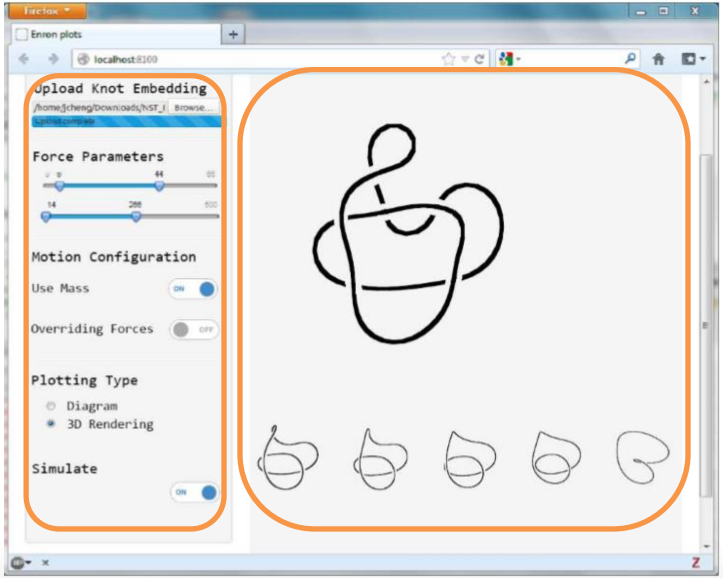

Figure 3. MathSimWeb.

System Architecture

R(HiPLAR)

- A back-end module

```
library(Matrix);
n <- 5000;
system.time(KnotSim(n));
# user  system elapsed
# 107.90  0.356  108.59
library(HiPLARM)
system.time(KnotSim(n));
# user  system elapsed
# 21.012  0.316  22.34
```

V Conclusion

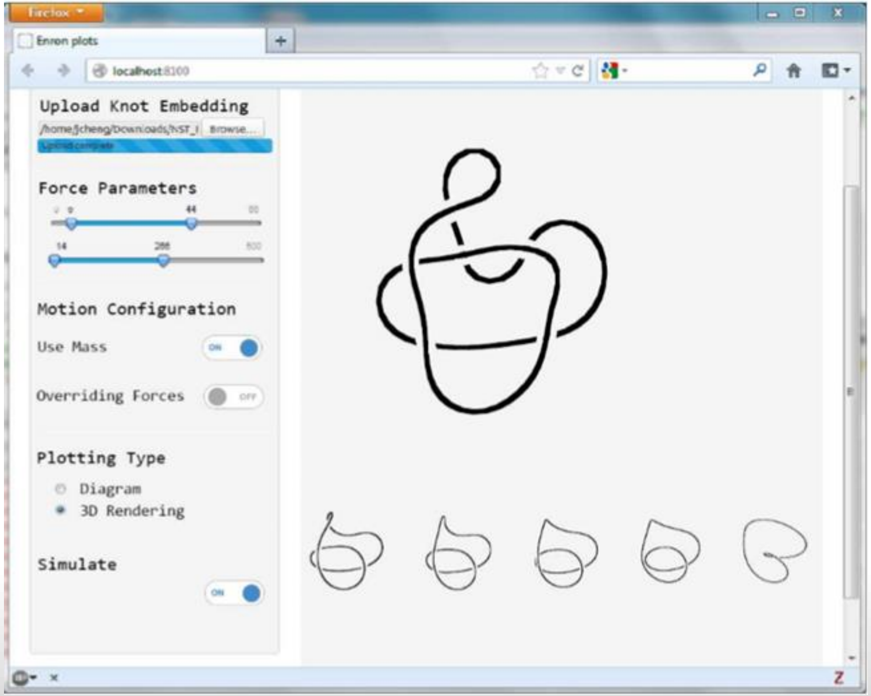

Figure 3. MathSimWeb.

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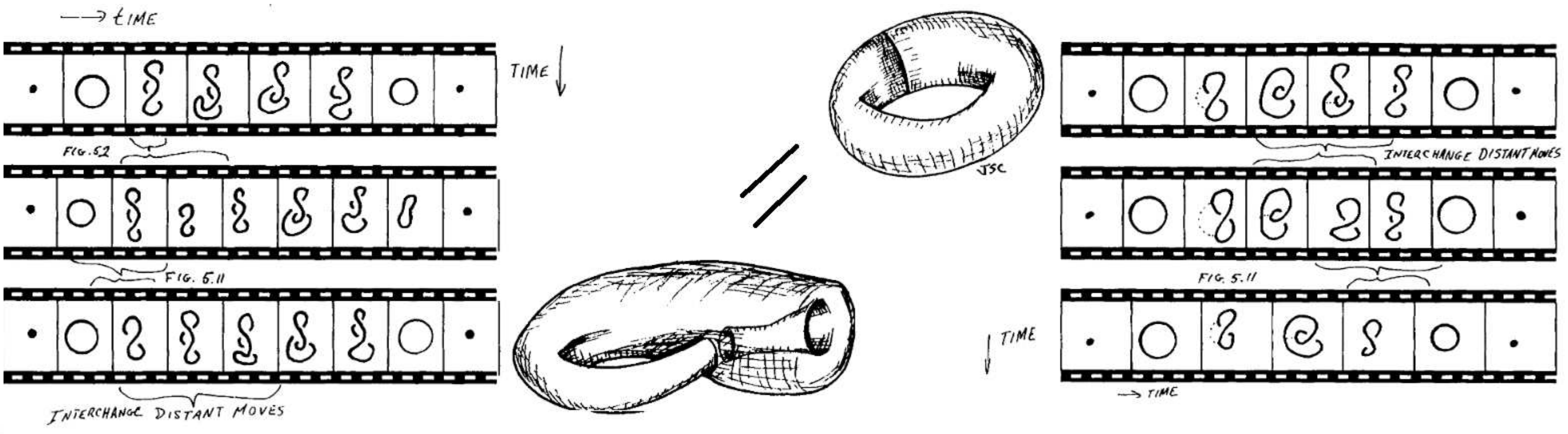

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